$$q_w = \frac{(K'''_{\mu w} + 1)(I_{C'_1} - I_{C'''})}{(x_{B''} - x_{B'}) + \mu(w'' - w')} .$$

The method given above can be used to design installations for drying with a gas suspension containing hygroscopic particles and to determine the heat, air, and dispersed material consumed in drying.

NOTATION

c, specific mass heat capacity, kJ/(kg·°C); G, mass flow rate, kg/sec; I, enthalpy per unit mass, Kj/kg; Q, heat, kW; t, temperature, °C; w, moisture content per dry mass, fraction; x, moisture content per unit mass of dry air, kg/kg; μ, specific mass flow-rate concentration of particles, kg/kg of dry air; φ, relative humidity, fraction; r, heat of vaporization of water, kJ/kg. Indices: w, water; v, vapor; a, air; p, particles; ap, air substituting for particles; gs, gas supension; e, equilibrium; b, bound; $^{-}$, average; without primes, at entrance to air chamber; ''', at exit from air chamber; ', at entrance to drying chamber; '', at exit from drying chamber; 1-4, intermediate parameters.

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KINETICS OF ISOTHERMAL EVAPORATION OF A POROUS OR DISPERSED BODY

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We consider the evaporation of a porous body which consists initially of spheres of identical radii. We find the dependence of the velocity of motion of the boundary of the body on the external parameters.

DERIVATION OF EQUATIONS OF MOTION FOR THE BOUNDARY

To simplify the calculations, we confine ourselves in the present work to a model of the porous body which consists of spheres of identical radii r_0 which are positioned at random but uniformly to the right of a plane. The evaporation takes place on account of a difference between the pressure of vapor in equilibrium with the spheres p_0, and the pressure p_1 of the vapor confined on the left of the planar boundary of the body. Below, we neglect the dependence of the equilibrium pressure on the radius of the spheres. After the beginning of evaporation, the radii of spheres are no longer equal. The radii of the spheres which are nearer to the periphery of the body (Fig. 1) decrease more rapidly, and a gradient of radii appears (but not the gradient of concentrations of the spheres) which is directed and decays toward the interior of the porous body. The spheres at the external surface of the body evaporate first.

We consider a steady-state evaporation process during which the external boundary of a dispersed body which corresponds to radii of spheres $r = 0$ moves to the right with a constant, but yet unknown, velocity c. The problem consists of determination of c as a func-

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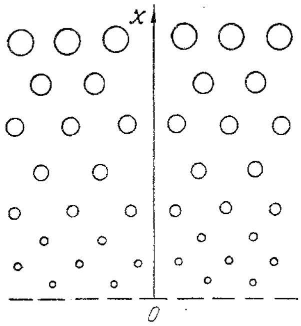

Fig. 1. Model of evaporating porous body used in this work.

tion of the parameters p_0, p_1, r_0, and n (their number per unit volume of the dispersed body). The resistance of the spheres to vapor transport is neglected.

We introduce a moving coordinate system whose origin x = 0 coincides with the external boundary of the body which moves to the right with a constant velocity c. It is clear that the mass of the substance (including the mass of the spheres and the vapor between them) in the volume between two planes with arbitrary coordinates x_1 and x_2, remains constant. Hence it follows that the current of matter per unit area through any such plane is identical and equal to the intensity of evaporation:

$$j = cn\rho_s V + c\rho_v(1 - nV) + D\frac{d\rho_v}{dx} = \text{const}, \tag{1}$$

where $V = \frac{4}{3}\pi r^3$ is the volume of a sphere. Using the equation of state of an ideal gas $\rho_v = p\frac{\mu}{RT}$, Eq. (1) will be written as

$$j = cn\rho_s\frac{4}{3}\pi r^3 + \frac{cp\mu}{RT}\left(1 - n\frac{4}{3}\pi r^3\right) + \frac{D\mu}{RT}\frac{dp}{dx} = \text{const}. \tag{2}$$

The boundary conditions for Eq. (2) are

$$p \to p_0,\ x \to \infty;\ p = p_1,\ x = 0. \tag{2'}$$

In (1) and (2), the diffusion coefficient of the vapor is determined as follows. We assume that the following relation holds:

$$\lambda \gg l = \frac{4}{S} = \frac{1}{\pi n r^2}, \tag{3}$$

Then, the Knudsen molecular regime of gas flow is realized in the pores for which we have, according to [1],

$$D = \alpha\frac{\varepsilon}{S} = \frac{\alpha(1 - nV)^2}{S}. \tag{4}$$

If also the reflection is diffusive with total accomodation, we have

$$\alpha = \frac{12}{13}\left(\frac{8RT}{\pi\mu}\right)^{1/2}. \tag{5}$$

Near the boundary of the disperse body ($x \to 0$), condition (3) is violated because r is small. However, the following condition begins to hold:

$$l \gg \lambda \gg r, \tag{6}$$

which corresponds to pseudomolecular gas flow discovered earlier in [2]. In this case, however, the "diffusion" coefficient is increased insignificantly by 10% from the value (4) which will be neglected.

From the condition of steady-state evaporation follows the equation

$$\frac{dV}{dt}=\frac{\partial V}{\partial t}+c\frac{\partial V}{\partial x}=0, \tag{7}$$

where dV/dt is the derivative in the moving coordinate system, and ∂V/∂x in the stationary system. When condition (6) holds, and condition (3) holds with a lesser but sufficient accuracy, the rate of evaporation ∂V/∂t is equal to

$$\frac{\partial V}{\partial t}=\beta S_0(p_0-p)=\beta 4\pi r^2(p_0-p), \tag{8}$$

where the quantity β is defined as [2]:

$$\beta=\left(\frac{\mu}{2\pi RT}\right)^{1/2}\frac{1}{\rho_s}. \tag{9}$$

It follows from relations (7) and (8) that

$$\frac{\partial V}{\partial x}=\frac{1}{c}\beta 4\pi r^2(p_0-p), \tag{10}$$

or

$$p=p_0-\frac{c}{\beta 4\pi r^2}\frac{\partial V}{\partial x}=p_0-\frac{c}{\beta}\frac{dr}{dx}, \tag{11}$$

Hence,

$$\frac{dp}{dx}=-\frac{c}{\beta}\frac{d^2r}{dx^2}.$$

We substitute the obtained relations in the original equation (2):

$$cn\rho_s\frac{4}{3}\pi r^3+\frac{c\mu p}{RT}\left(1-n\frac{4}{3}\pi r^3\right)+\frac{\alpha\mu\left(1-n\frac{4}{3}\pi r^3\right)^2}{RT4\pi nr^2}\frac{dp}{dx}=\text{const}, \tag{12}$$

which gives

$$cn\rho_s\frac{4}{3}\pi r^3+\frac{c\mu}{RT}\left(p_0-\frac{c}{\beta}\frac{dr}{dx}\right)\left(1-\frac{4}{3}\pi nr^3\right)-\frac{\alpha\mu c\left(1-\frac{4}{3}\pi nr^3\right)^2}{\beta RT4\pi nr^2}\frac{d^2r}{dx^2}=\text{const}. \tag{13}$$

When going away from the boundary of the dispersed body toward the interior, i.e. for $x\to\infty$, we have $\frac{dp}{dx}\to 0,\ r\to r_0$, and we therefore obtain from (12)

$$\text{const}=c\rho_s\frac{4}{3}\pi nr_0^3+\frac{c\mu p_0}{RT}\left(1-\frac{4}{3}\pi nr_0^3\right), \tag{14}$$

This determines the constant in (12) and (13). We substitute the obtained constant in Eq. (13) which gives, after simple transformations,

$$\frac{\mu\alpha}{\beta RT}\frac{d^2r}{dx^2}+\frac{c\mu}{\beta RT}\frac{4\pi nr^2}{\left(1-\frac{4}{3}\pi nr^3\right)}\frac{dr}{dx}=\frac{\frac{16}{3}\rho_s\pi^2n^2r^2(r^3-r_0^3)}{\left(1-\frac{4}{3}\pi nr^3\right)^2}. \tag{15}$$

When integrating Eq. (15), one must satisfy the following boundary conditions which follow from conditions (2'):

$$r=0 \text{ for } x=0, \tag{16}$$

$$r\to r_0 \text{ for } x\to\infty, \tag{17}$$

$$\frac{c}{\beta}\frac{dr}{dx}=p_0-p_1 \text{ for } x=0. \tag{18}$$

SOLUTION OF THE FUNDAMENTAL EQUATION

We now obtain the solution of Eq. (15) with boundary conditions (16)-(18). The aim is to determine the unknown velocity of motion of the boundary c. For convenience we introduce in (15)-(18) the dimensionless quantities

$$y = x/x_0,\ z = c/c_0,\ \xi = r/r_0,\ \varkappa = \frac{4}{3}\pi n r_0^3, \tag{19}$$

$$x_0 = \frac{1}{4\pi n r_0^2}\sqrt{\frac{3\mu\alpha}{\rho_s\beta RT}},$$

$$c_0 = \sqrt{\frac{\beta\rho_s\alpha RT}{3\mu}}, \tag{20}$$

$$\gamma = \frac{(p_0 - p_1)\mu}{\varkappa\rho_s RT}. \tag{21}$$

The quantity ξ is a function of the coordinate y, and $0 \leqslant \xi \leqslant 1$. With the dimensionless quantities (19)-(21), the problem (15)-(18) can be rewritten as

$$\xi'' + \frac{z\xi^2\xi'}{1-\varkappa\xi^3} = -\frac{\xi^2(1-\xi^3)}{(1-\varkappa\xi^3)^2}, \tag{22}$$

$$\xi(0) = 0, \tag{23}$$

$$\xi(y) \to 1,\ y \to \infty, \tag{24}$$

$$\xi'(0) = \frac{\gamma}{z}. \tag{25}$$

Hence it follows that the dimensionless velocity z is a function of only two parameters $\varkappa$ and γ, and in the case $\varkappa \ll 1$, the system (22)-(25) can be written as

$$\xi'' + z\xi^2\xi' = -\xi^2(1-\xi^3),$$

$$\xi(0) = 0,$$

$$\xi(y) \to 1,\ y \to \infty,\ \xi'(0) = \frac{\gamma}{z}.$$

Hence it follows that in this case, z is a function of only one parameter γ. Since (22) does not contain explicitly the independent variable y, one can make the substitution $\xi' = f(\xi^3)$ and, after some transformations, this yields an equation of the form

$$3f\frac{df}{d\tau} + \frac{zf}{1-\varkappa\tau} = -\frac{1-\tau}{(1-\varkappa\tau)^2}, \tag{26}$$

where $\tau = \xi^3$. Putting in (26) $u = f^2$ or $f = \sqrt{u}$, we obtain the following first-order equation:

$$\frac{3}{2}u' + \frac{z\sqrt{u}}{1-\varkappa\tau} = \frac{1-\tau}{(1-\varkappa\tau)^2},\ 0 \leqslant \tau \leqslant 1, \tag{27}$$

with the boundary conditions

$$u(0) = \frac{\gamma^2}{z^2}, \tag{28}$$

$$u(1) = 0. \tag{29}$$

The boundary condition (29) follows from the fact that $\xi'(y) \to 0$ for $y \to \infty$.

A numerical solution of the problem (27)-(29) can easily be obtained on a computer. It can, however, be also solved analytically in the case of small and large values of z.

The Case $z \ll 1$. In this case, instead of Eq. (27) we have

$$\frac{3}{2}u' = -\frac{1-\tau}{(1-\varkappa\tau)^2}, \tag{30}$$

which allows simple integration:

$$u(\tau) = -\frac{2}{3}\int_0^\tau \frac{1-\tau}{(1-\varkappa\tau)^2}\,d\tau + \frac{\gamma^2}{z^2}. \tag{31}$$

In obtaining (31) we used the boundary condition (28). Carrying out the integration in (31) we have

$$u(\tau) = \frac{2}{3}\,\frac{1-\varkappa}{\varkappa}\,\frac{\tau}{1-\varkappa\tau} + \frac{2}{3\varkappa^2}\ln(1-\varkappa\tau) + \frac{\gamma^2}{z^2}. \tag{32}$$

From expression (32) and the boundary conditions (29) we find an equation for the dependence of z on γ and $\varkappa$:

$$\frac{2}{3\varkappa} + \frac{2}{3\varkappa^2}\ln(1-\varkappa) + \frac{\gamma^2}{z^2} = 0. \tag{33}$$

Hence,

$$z = \frac{\gamma\sqrt{\frac{3}{2}\varkappa}}{\sqrt{\frac{1}{\varkappa}\ln\frac{1}{1-\varkappa} - 1}}.$$

In the case $\varkappa \ll 1$, expression (33) yields, by expanding the logarithm up to the second term,

$$z \approx \sqrt{3}\gamma, \tag{34}$$

or, in the dimensional form, by substituting expressions (20) and (21),

$$c \approx \frac{p_0 - p_1}{\frac{4}{3}\pi n r_0^3}\sqrt{\frac{\alpha\beta\mu}{\rho_s RT}}. \tag{34'}$$

The condition of applicability of the obtained expressions is

$$\gamma \ll 1. \tag{35}$$

By definition of the quantity γ (21), we hence find

$$\frac{p_0(1-p_1/p_0)\,\mu}{RT}\,\frac{1}{\rho_s}\,\frac{1}{\varkappa} \ll 1. \tag{36}$$

But $\frac{p_0\mu}{RT} = \rho_v^0$ is the equilibrium pressure of vapor above the particles. Since also $p_0/p_1 > 1$, the last inequality will clearly be satisfied if

$$\frac{\rho_v^0}{\rho_s}\,\frac{1}{\varkappa} \ll 1 \tag{37}$$

or

$$\rho_v^0/\rho_s \ll \varkappa.$$

Since usually $\rho_v^0/\rho_s \sim 10^{-3}$–$10^{-4}$, and $\varkappa \sim 1 - 10^{-2}$, condition (37) is in most cases satisfied to a high degree of accuracy.

We estimate in this case the thickness of the particle layer δ where the particle radius differs from the equilibrium radius r_0. We denote the dimensionless thickness by $\Delta = \delta/x_0$. We make this estimate in the case $\varkappa \ll 1$. Then, by noting that $z = \sqrt{3}\gamma$, we obtain from expression (32)

$$u(\tau) \approx \frac{(\tau-1)^2}{3}.$$

Since $u = f^2 = (\xi')^2$,

$$\xi_y' = -\frac{1-\xi^3}{\sqrt{3}}. \tag{38}$$

For $y \to \infty$, the last equation can be written in the form

$$\xi_y' \approx -\sqrt{3}(1-\xi),$$

whose solution is

$$\xi = 1 - A e^{-\sqrt{3}y}, \tag{39}$$

where the constant A can be obtained by joining the expressions for $\xi(y)$ for small and large values of the argument y. From (39) we obtain

$$\Delta \sim \frac{1}{\sqrt{3}} \tag{40}$$

or, in the dimensional form,

$$\delta \sim \frac{1}{4\pi n r_0^2} \sqrt{\frac{\mu\alpha}{\rho_s \beta R T}}. \tag{40'}$$

The Case $z \gg 1$. By making the transformation of variables $\varphi = z\tau$, in Eq. (27), we obtain

$$\frac{3}{2} \frac{du}{d\varphi} + \frac{\sqrt{u}}{1 - \frac{\varkappa\varphi}{z}} = -\frac{1}{z} \frac{1 - \frac{\varphi}{z}}{\left(1 - \frac{\varkappa}{z}\varphi\right)^2}. \tag{41}$$

The term on the right-hand side is small because of the assumption $z \gg 1$. This makes it possible to write Eq. (41) in the form

$$\frac{3}{2} \frac{du}{d\varphi} + \frac{\sqrt{u}}{1 - \frac{\varkappa\varphi}{z}} = 0, \quad 0 \leqslant \varphi \leqslant z, \tag{42}$$

$$u(0) = \frac{\gamma^2}{z^2}, \tag{43}$$

$$u(z) = 1. \tag{44}$$

Integrating (42) and using the boundary condition (43), we find

$$u(\varphi) = \left[\frac{z}{3\varkappa} \ln\left(1 - \frac{\varkappa}{z}\varphi\right) + \frac{\gamma}{z}\right]^2. \tag{45}$$

Substituting $\varphi = z$, in Eq. (45), we obtain from the boundary condition (44):

$$\frac{z}{3\varkappa} \ln(1-\varkappa) + \frac{\gamma}{z} = 0,$$

which gives the following expression for z:

$$z = \sqrt{\frac{3\gamma\varkappa}{\ln\frac{1}{1-\varkappa}}}. \tag{46}$$

The condition of applicability of the last expressions is $\gamma \gg 1$ or, as follows from (37),

$$\frac{\rho_v^0}{\rho_s}\left(1 - \frac{p_1}{p_0}\right) \gg \varkappa. \tag{47}$$

It follows from expression (47) that this approximation is applicable only for very small $\varkappa \leqslant 10^{-4}$, i.e., for a very dilute cloud of particles.

As in the previous case, we estimate the quantity Δ by putting $\varkappa \ll 1$. We find from (45), by keeping only the first term in the expansion of the logarithm,

$$u = \frac{\gamma}{3}(1-\tau)^2.$$

Here we noted that $z = \sqrt{3\gamma}$. Since $\tau = \xi^3$, $u = (\xi')^2$, we have

$$\xi' = -\sqrt{\frac{\gamma}{3}}\,(1-\xi^3).$$

As before, for $y \to \infty$ we obtain from the last equation

$$\xi' \approx -\sqrt{3\gamma}\,(1-\xi), \tag{48}$$

whose solution has the form

$$\xi = 1 - B\,e^{-\sqrt{3\gamma}\,y}, \tag{49}$$

where the meaning of the constant B is analogous to that of the constant A in Eq. (39). From expression (49) we obtain

$$\Delta \sim \frac{1}{\sqrt{3\gamma}}, \tag{50}$$

or, in the dimensional form,

$$\delta \sim \sqrt{\frac{\alpha}{12\,\beta\pi n r_0 (p_0 - p_1)}}. \tag{50'}$$

By comparing expressions (40) and (50) we obtain that for small velocities of motion, the width of the front is considerably larger than at high velocities since in the latter case $\gamma \gg 1$.

Thus, the dependence $z(\gamma)$ for $\varkappa \ll 1$ has the form shown in Fig. 2.

The presented theory can be used in the calculation of evaporation of many dispersed and porous media such as porous ablating coverings and the evaporation of clouds and domets. In some cases, the mass evaporation flow from a unit surface of a porous body exceeds the mass flow from a unit continuous surface. If the heat flux incident on the surface of the porous body is constant, the protective properties of the porous body will be higher than of a continuous body.

For the convenience of practical applications we shall go over from the dimensionless quantities to dimensional ones.

It follows from formula (14) that

$$j = c\,[\rho_s (1-\varepsilon) + \rho_v \varepsilon], \tag{51}$$

where $\varepsilon = 1 - \frac{4}{3}\pi n r_0^3$ is the porosity.

If the velocity c has been found, expression (51) makes it possible to determine the current j. For example, for $\gamma \ll 1$, we find from (51) and (34),

$$j = [\rho_s (1-\varepsilon) + \rho_v \varepsilon]\,\frac{p_0 - p_1}{1-\varepsilon}\sqrt{\frac{\mu\alpha\beta}{\rho_s RT}}$$

or, after the substitution $\alpha\beta = 24/(13\,\pi\rho_s)$,

$$j = [\rho_s (1-\varepsilon) + \rho_v \varepsilon]\,\frac{p_0 - p_1}{(1-\varepsilon)\,\rho_s}\sqrt{\frac{24\,\mu}{13\,\pi RT}}. \tag{52}$$

We denote

$$j_0 = (p_0 - p_1)\sqrt{\frac{\mu}{2\pi RT}}, \tag{53}$$

Then

$$j/j_0 = \left(1 + \frac{\varepsilon}{1-\varepsilon}\,\frac{\rho_v}{\rho_s}\right)\sqrt{\frac{48}{13}}.$$

Because of the assumption (37) $\rho_v/\rho_s(1-\varepsilon) \ll 1$, $\varepsilon < 1$ we obtain from (53)

$$j/j_0 = \sqrt{\frac{48}{13}}. \tag{54}$$

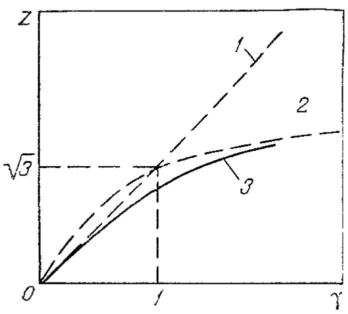

Fig. 2. Dependence of the dimensionless velocity of motion of the evaporation front z on the dimensionless parameter γ which characterizes the state of the system. 1) Approximate soln. $z = \sqrt{3\gamma}$; 2) approximate soln. $z = \sqrt{3\gamma}$; 3) exact solution.

In the other limiting case $\gamma \gg 1$, we obtain from (51) and (47)

$$j = [\rho_s(1-\varepsilon) + \rho_v\varepsilon] \sqrt{\frac{p_0 - p_1}{1-\varepsilon}} \sqrt{\frac{24}{13\pi\rho_s}}^{*} .$$

Hence we find, using (52),

$$j/j_0 = \sqrt{\frac{48}{13}} \left(\sqrt{\frac{\rho_s(1-\varepsilon)}{\rho_v}} + \varepsilon \sqrt{\frac{\rho_v}{\rho_s(1-\varepsilon)}} \right) \frac{1}{\sqrt{1-p_1/p_0}} .$$

Because of the assumption $\gamma \gg 1$: $\rho_s(1-\varepsilon)/\rho_v \ll 1$, i.e.,

$$j/j_0 = \sqrt{\frac{48}{13}}\, \varepsilon \sqrt{\frac{\rho_v}{\rho_s(1-\varepsilon)}} \frac{1}{\sqrt{1-p_1/p_0}} . \quad (55)$$

In the last expression, because of the assumptions $\rho_v/\rho_s(1-\varepsilon) \gg 1$, so that the quantity $1-\varepsilon$ is very small, i.e., the porosity is very close to unity.

NOTATION

ρ_s and ρ_v, densities of the material of the spheres and vapor; $\rho_v = \rho_v(p)$; μ, molar weight; R, gas constant; T, temperature in degrees K; D, diffusion coefficient of the vapor; λ, mean free path of the vapor molecules; $S = 4\pi n r^3$, specific surface of the porous body; l, mean free path of the vapor molecules between two collisions with the spheres; ε, porosity of the body; and S_0, surface of one granule.

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*Generally speaking, in the case of intense evaporation one can use another expression for the evaporation rate [3].